

The relation between $\Delta\nu$ and ν_{\max} for solar-like oscillations

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ABSTRACT

Establishing relations between global stellar parameters and asteroseismic quantities can help improve our understanding of stellar astrophysics and facilitate the interpretation of observations. We present an observed relation between the large frequency separation, $\Delta\nu$, and the frequency of maximum power, ν_{\max} . We find that $\Delta\nu \propto \nu_{\max}^{0.77}$, allowing prediction of $\Delta\nu$ to about 15 per cent given ν_{\max} . Our result is further supported by established scaling relations for $\Delta\nu$ and ν_{\max} and by extended stellar model calculations, which confirm that $\Delta\nu$ can be estimated using this relation for basically any star showing solar-like oscillations in the investigated range ($0.5 < M/M_{\odot} < 4.0$).

Key words: stars: fundamental parameters — stars: oscillations — stars: interiors.

1 INTRODUCTION

Simple scaling relations for asteroseismic quantities have proven very useful when analysing stellar oscillations. In particular, relations for scaling the quantities ν_{\max} (Brown et al. 1991) and $\Delta\nu$ (Ulrich 1986) from the Sun have been widely used to constrain stellar global parameters, or simply to verify the asteroseismic signal of stars that already had well-constrained stellar parameters. Here, ν_{\max} is the frequency of maximum power of the oscillations, and $\Delta\nu$ is the so-called large frequency separation between consecutive overtones. For the Sun, $\nu_{\max} \simeq 3100 \mu\text{Hz}$ and $\Delta\nu \simeq 135 \mu\text{Hz}$.

Over the past decade, detections of solar-like oscillations in other stars have been growing in number (see reviews by Aerts et al. 2008; Bedding & Kjeldsen 2008), and the *CoRoT* space mission has caused a recent surge (Michel et al. 2008; de Ridder et al. 2009). This flow of data is expected to increase to a flood with the *Kepler* mission (Christensen-Dalsgaard et al. 2008). For many stars, knowing $\Delta\nu$ or ν_{\max} can help constrain the stellar parameters considerably (Stello et al. 2008; Kallinger et al. 2008; Stello et al. 2009; Miglio et al. 2009) and hence yield important new insights into stellar structure and evolution, even without a full asteroseismic analysis. In addition, recent developments of automated software for asteroseismic data analysis benefit from knowing a simple relation between $\Delta\nu$ and ν_{\max} (Mathur et al. 2009; Huber et al. 2009; Hekker et al. 2009b).

In this Letter we investigate the relation between $\Delta\nu$ and ν_{\max} , building on previous studies to show how these two quantities are related to each other and to the global stellar parameters. We also carry out stellar model calcula-

tions to support this investigation, including a comparison of stellar models with the established scaling relations for $\Delta\nu$ and ν_{\max} .

2 THE OBSERVED RELATION

Table 1 shows published measurements of $\Delta\nu$ and ν_{\max} for 55 stars including the Sun. In some cases, where the value of ν_{\max} was not specified, we have estimated it from the published power spectra. The location of the stars in the *H-R* diagram are shown in Fig. 1.

In Fig. 2 we plot $\Delta\nu$ versus ν_{\max} for the data listed in Table 1. We see a remarkably tight relation over nearly three orders of magnitude. A power-law fit gives

$$\Delta\nu = (0.263 \pm 0.009)\mu\text{Hz} (\nu_{\max}/\mu\text{Hz})^{0.772 \pm 0.005}. \quad (1)$$

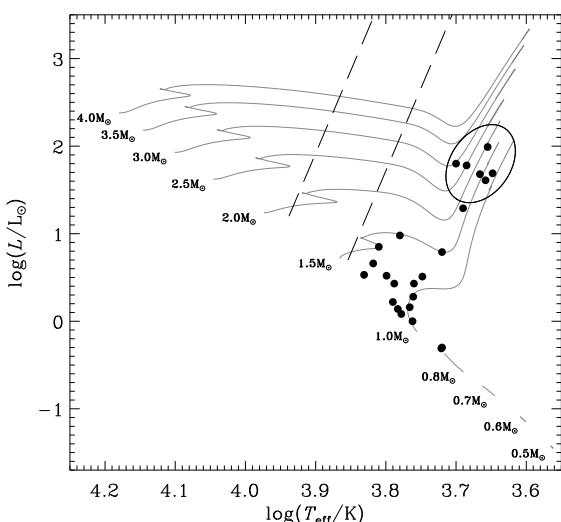
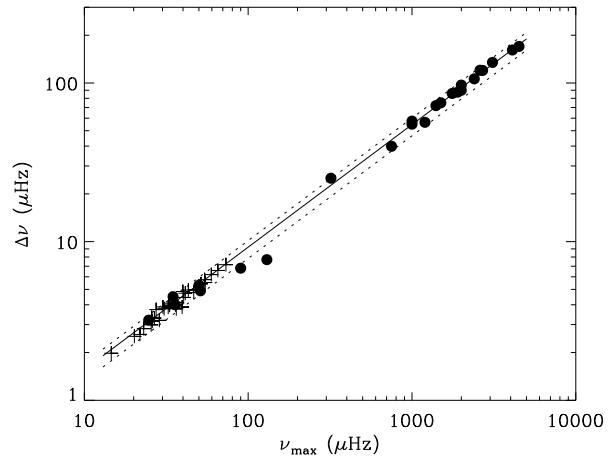
The two stars that deviate the most from this relation are η Ser and ξ Hya. In both cases, there are reported ambiguities in the determination of the large separation, which could explain the deviation. Apart from those two cases, all stars in Fig. 2 fall within +10 per cent and -15 per cent of the fitted relation, while the main sequence stars alone fall within ± 5 per cent. We note that Hekker et al. (2009b) have confirmed the tight correlation for a sample of several hundred red giants observed by *CoRoT*, which also seems to indicate a tendency for some stars to fall below the relation.

3 SCALING RELATIONS

Can previously published scaling relations for $\Delta\nu$ and ν_{\max} explain the observed relation between them? It is well-established (e.g. Ulrich 1986) that, to a good approximation,

Table 1. Published measurements of $\Delta\nu$ and ν_{\max}

Star	$\Delta\nu$ (μHz)	ν_{\max} (μHz)	Source
τ Cet	170	4500	Teixeira et al. (2009)
α Cen B	161.4	4100	Kjeldsen et al. (2008)
Sun	134.8	3100	Kjeldsen et al. (2008)
ι Hor	120	2700	Vauclair et al. (2008)
γ Pav	120.3	2600	Mosser et al. (2008)
α Cen A	106.2	2400	Kjeldsen et al. (2008)
HD175726	97	2000	Garcia et al. (2009)
μ Ara	90	2000	Bouchy et al. (2005)
HD181906	87.5	1900	Garcia et al. (2009)
HD49933	85.9	1760	Garcia et al. (2009)
HD181420	75	1500	Garcia et al. (2009)
β Vir	72	1400	Carrier et al. (2005b)
μ Her	56.5	1200	Bonanno et al. (2008)
β Hyi	57.5	1000	Kjeldsen et al. (2008)
Procyon	55	1000	Arentoft et al. (2008)
η Boo	39.9	750	Carrier et al. (2005a)
ν Ind	25.1	320	Kjeldsen et al. (2008)
η Ser	7.7	130	Barban et al. (2004)
ξ Hya	6.8	90	Frandsen et al. (2002)
β Vol	4.9	51	unpublished WIRE data
ϵ Oph	5.3	50	Barban et al. (2007)
ξ Dra	4.0	36	unpublished WIRE data
κ Oph	4.5	35	unpublished WIRE data
HR3280	3.2	25	unpublished WIRE data
31 <i>CoRoT</i> giants	2–7	15–73	Kallinger et al. (2008)

**Figure 1.** *H*–*R* diagram showing all stars listed in Table 1. The ellipse indicates the approximate location of the 31 *CoRoT* red giants. The evolutionary tracks (grey curves), illustrate the range in mass and evolutionary state that we investigate with our stellar models (see Sect. 4). The dashed lines show the approximate location of the classical instability strip (Saio & Gautschy 1998).**Figure 2.** Observed $\Delta\nu$ versus ν_{\max} for the stars listed in Table 1. The 31 *CoRoT* red giants are plotted with plus symbols. The solid line is a power-law fit, and dotted lines show +10 per cent and -15 per cent deviations.

$\Delta\nu$ is proportional to the square root of the stellar density:

$$\frac{\Delta\nu}{\Delta\nu_{\odot}} = \sqrt{\frac{\rho}{\rho_{\odot}}} = \frac{(M/M_{\odot})^{0.5}(T_{\text{eff}}/T_{\text{eff},\odot})^3}{(L/L_{\odot})^{0.75}}. \quad (2)$$

Also, following Brown et al. (1991) and Kjeldsen & Bedding (1995), we expect ν_{\max} to scale as the acoustic cut-off frequency, ν_{ac} . Hence,

$$\frac{\nu_{\max}}{\nu_{\max,\odot}} = \frac{\nu_{\text{ac}}}{\nu_{\text{ac},\odot}} = \frac{M/M_{\odot}(T_{\text{eff}}/T_{\text{eff},\odot})^{3.5}}{L/L_{\odot}}, \quad (3)$$

where it is observed for the Sun that $\nu_{\text{ac},\odot} \simeq 1.7\nu_{\max,\odot}$ (Balmforth & Gough 1990; Fossat et al. 1992). Given that $\Delta\nu$ and ν_{\max} scale differently with stellar parameters, the tightness of the correlation in Fig. 2 is perhaps surprising. To understand this, we raise Eq. (3) to the power a and divide by Eq. (2) which, after rearranging, gives

$$\frac{\Delta\nu}{\Delta\nu_{\odot}} = \left[\frac{(M/M_{\odot})^{0.5-a}(T_{\text{eff}}/T_{\text{eff},\odot})^{3-3.5a}}{(L/L_{\odot})^{0.75-a}} \right] \left(\frac{\nu_{\max}}{\nu_{\max,\odot}} \right)^a. \quad (4)$$

The tight relation in Fig. 2 leads us to conclude that the scaling factor in square brackets in Eq. (4) must be approximately constant when $a = 0.77$. Indeed, this is easy to see because that value of a eliminates almost completely the parameter that varies the most over the stars being considered, namely L , and leaves a very weak dependence on M and T_{eff} . This is illustrated in Fig. 3, which shows the scaling factor for the observed value $a = 0.77$. The scaling factor indeed covers a band 25 per cent wide for the region sampled by the stars (see Fig. 1), in agreement with the +10 to -15 per cent deviation shown by the observational data in Fig. 2. Figure 3 further supports the lower scatter observed for the main sequence stars and the higher, slightly skewed, scatter for red giants. We have determined the value of a that minimizes the spread in the scaling factor and found that it depends slightly on the stellar parameter range that we consider. For example, consideration of the entire model grid requires $a = 0.78$, while considering only ZAMS models gives $a = 0.75$.

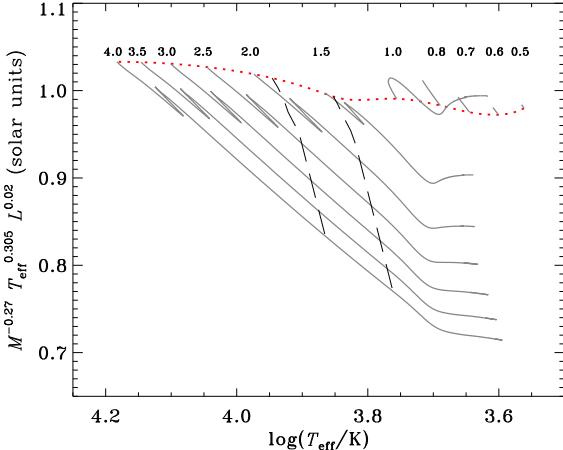


Figure 3. The scaling factor in square brackets in Eq. (4) for $a = 0.77$, for the same evolutionary tracks as in Fig. 1. The horizontal dotted curve is the ZAMS and the dashed lines indicate the instability strip. Masses are indicated in solar units.

4 MODELS

We now expand our investigation of the $\Delta\nu-\nu_{\max}$ relation using two sets of stellar models. One is a dense grid, comprising over a million models derived using the Aarhus stellar evolution code ASTEC (Christensen-Dalsgaard 2008b) and the adiabatic pulsation code ADIPLS (Christensen-Dalsgaard 2008a). This grid is slightly expanded in parameter space but otherwise identical to the one constructed by Stello et al. (2009) (see references herein). The other is a set derived using the Yale code YREC (Demarque et al. 2008) with mixing-length parameter $\alpha = 1.80$, initial hydrogen abundance $X = 0.707$, and metallicity $Z = 0.018$. Although the set of YREC models is more sparse, with a total of 92 models, it spans almost uniformly the same range in parameter space as the ASTEC grid. The region covered by the models is indicated by the evolutionary tracks in Fig. 1.

In addition to the global stellar parameters M , L , and T_{eff} , we calculated for each model the large frequency separation as the inverse of the sound travel time through the star:

$$\Delta\nu = \left[2 \int_0^R c^{-1} dr \right]^{-1}, \quad (5)$$

where c is the sound speed (Tassoul 1980; Gough 1986). We also calculated the acoustic cutoff frequency by assuming an isothermal atmosphere, which gives (Balmforth & Gough 1990)

$$\nu_{\text{ac}} = \frac{c}{2H}, \quad (6)$$

evaluated at the surface ($T = T_{\text{eff}}$). Here, $H = p/(g\rho)$ is the density scale height, p is pressure, g is gravity, and ρ is density.

In Fig. 4 we show a subset of the ASTEC grid for a fixed metallicity ($Z = 0.014$). Contours of constant $[2 \int dr/c]^{-1}$ (magenta) are almost parallel to contours of constant $c/2H$ (cyan), indicating a strong correlation. Hence, knowing one of these two quantities gives a good estimate of the other. This is particularly pronounced in the lower-right corner of

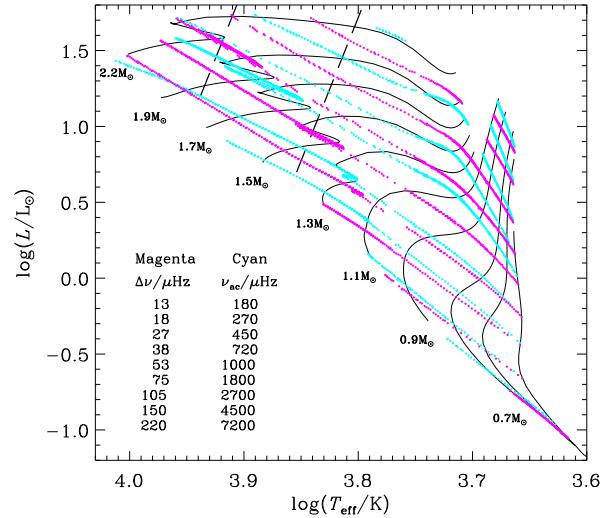


Figure 4. H - R diagram of grid subset (ASTEC models with $Z = 0.014$). Models within a narrow range of fixed values of $[2 \int dr/c]^{-1}$ (magenta) and $c/2H$ (cyan) are indicated. The dashed lines indicate the instability strip. Note that the low-mass models have been evolved beyond the age of the universe.

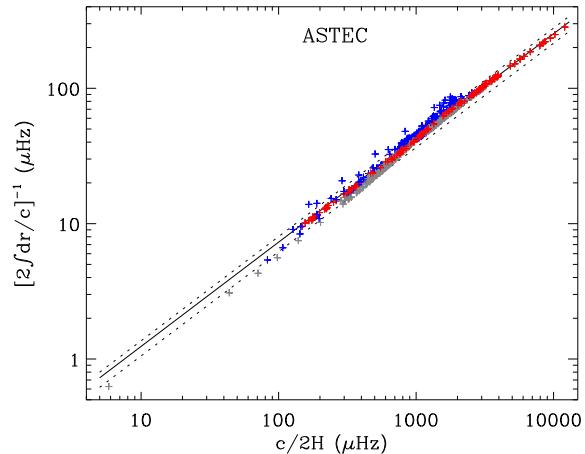


Figure 5. $\Delta\nu$ versus ν_{ac} relation for the ASTEC models. Blue symbols are models hotter than the red edge of the instability strip, while red and grey symbols are cooler. Red symbols are low mass ($M < 1.2M_{\odot}$). Solid line is a power-law fit and dotted lines show +10 per cent and -15 per cent deviations.

the diagram, corresponding to cool main-sequence and sub-giant stars. A somewhat weaker correlation is seen in the red giant phase. Note that we neglected the post He-core burning phase, including any mass loss associated with red giant branch evolution. If included, it would blur the contours in this region (Stello et al. 2008).

Figure 5 shows the relation between $\Delta\nu$ and ν_{ac} for the ASTEC models. We find very similar results for the YREC models. Stars hotter than the red edge of the instability strip are generally not expected to exhibit solar-like oscillations and are shown in blue, while cool models are shown in red and grey. We compare the models with observations by fit-

ting a power law with the observed value of $a = 0.77$ fixed. We see excellent agreement with observations, particularly at low masses ($< 1.2 M_{\odot}$, red), which is perhaps not surprising since the observations are predominantly from low-mass stars. Interestingly, all cool main-sequence models – high values of ν_{ac} – show a very tight relation, while for more evolved models – lower values of ν_{ac} – the higher mass ones (grey) scatter more and fall below the power-law relation. This agrees with the observations and with the scaling factor in Eq. (4) (see also Fig. 3). Finally, we found that the theoretical $\Delta\nu-\nu_{\text{ac}}$ relation was not notably sensitive to metallicity. However, it remains to be explained why $\Delta\nu$, which depends on the stellar mean density, correlates so strongly with ν_{ac} , which depends on the local conditions in the atmosphere.

5 DISCUSSION

Several points should be kept in mind when comparing the model calculations with the observations. Firstly, we do not measure ν_{ac} directly in stars and instead rely on its relation to ν_{max} (see Eq. 3), assuming that $\nu_{\text{max}}/\nu_{\text{ac}}$ does not vary from the solar value. In addition, ν_{max} can be difficult to define and measure, especially for stars with broad or double-humped envelopes, as seen in Procyon (Bedding & Kjeldsen 2006; Arentoft et al. 2008). On the other hand, $\Delta\nu$ varies with frequency and mode degree, and its measurement from the power spectrum gives an average that will not be exactly equal to $[2 \int dr/c]^{-1}$. That said, Fig. 5 clearly supports the tight $\Delta\nu-\nu_{\text{max}}$ relation that we observe.

The preceding comments relate to measuring $\Delta\nu$ and ν_{max} from observations. We should also consider the various ways in which these quantities can be estimated from the models. So far we have used Eqs. (5) and (6). Here, we derive the large frequency separation and the acoustic cut-off frequency from the models in other ways and investigate any significant systematic differences. The large separation was derived by fitting to 11 consecutive radial orders around ν_{max} , which we denote $\Delta\nu_{\text{fit}}$. The result for the YREC models (Fig. 6, left panel) shows agreement within a few per cent between $[2 \int dr/c]^{-1}$ and $\Delta\nu_{\text{fit}}$, although we note that the ratio $[2 \int dr/c]^{-1}/\Delta\nu_{\text{fit}}$ is always higher than unity. We see similar results for the ASTEC models.

We then tested two additional ways to calculate the acoustic cutoff frequency. Firstly, H was derived from the actual density gradient in the model, but still using Eq. (6). Secondly we used the full expression $\nu_{\text{ac}} = (c/2H)\sqrt{1 - 2dH/dr}$ (Balmforth & Gough 1990). The results are shown in Fig. 6 (right panel). Apart from a few hot models that deviate by up to 25 per cent, the agreement is again within a few per cent. For our purpose of investigating the relation between $\Delta\nu$ and ν_{ac} , a few per cent difference in either quantity is not important, but they should be kept in mind for other applications.

Finally, it is interesting to investigate how well the scaling relations for $\Delta\nu$ and ν_{ac} (Eqs. 2 and 3) agree with the model calculations (Eqs. 5 and 6). Figure 7 examines how well $[2 \int dr/c]^{-1}$ scales with $\sqrt{\rho}$. For cool models the agreement is particularly good, almost independent of the evolutionary state, while it deteriorates slightly for hot models (those below the blue line). However, we note

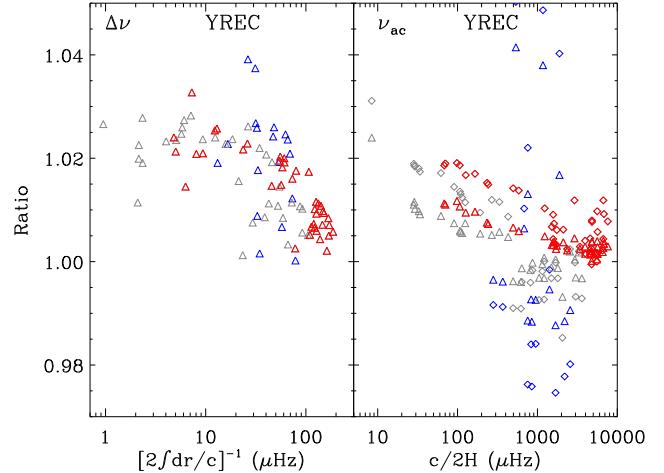


Figure 6. Left panel: Ratio between $[2 \int dr/c]^{-1}$ and a linear fit to model frequencies, $\Delta\nu_{\text{fit}}$. Right panel: Ratio between $c/2H_2$ and $c/2H_1$ where $H_2 = -dr/d\ln\rho$ and $H_1 = p/(g\rho)$ (triangles), and between $(c/2H_1)\sqrt{1 - 2dH/dr}$ and $c/2H_1$ (diamonds). Both panels show results for YREC models. Colour notation follows that of Fig. 5.

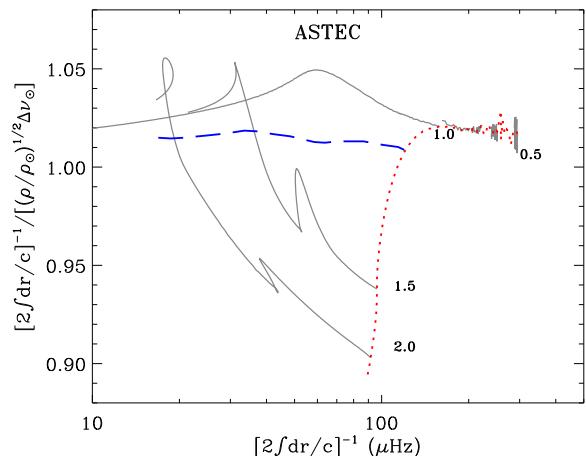


Figure 7. Ratio of $\Delta\nu$ between model calculations and solar scaling for ASTEC grid ($Z = 0.014$). Blue dashed curve shows models with $T_{\text{eff}} = 6400$ K. Annotation follows that of Fig. 3.

that $[2 \int dr/c]^{-1}$ systematically overestimates the large frequency separation for cool models, corresponding to 2–3 per cent for the Sun.

In a similar way Fig. 8 shows that the model calculations of $c/2H$ follow the $MT_{\text{eff}}^{3.5}/L$ scaling from the Sun quite well. The ratio is always below unity because we evaluate $c/2H$ at the surface defined as $T = T_{\text{eff}}$, where $c/2H$ is slightly below its maximum value, which occurs further out in the atmosphere. As with $\Delta\nu$, we conclude that the best agreement is for the cool models. We find similar results for the YREC models.

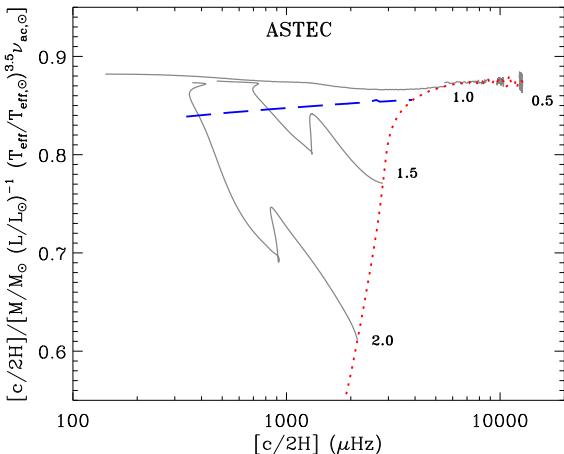


Figure 8. Ratio of ν_{ac} between model calculations and scaling for ASTEC grid ($Z = 0.014$). Annotation follows that of Fig. 7.

6 CONCLUSIONS

This Letter points out that the ratio $\Delta\nu/\nu_{\max}^{0.77}$ varies very weakly with stellar parameters, so that it is essentially constant. Hence, if either one of these parameters is measured, this gives a very useful and robust estimate of the other without any prior knowledge of the stellar global parameters, L , M , T_{eff} , and Z . We anticipate this relation can be used to establish the most plausible large separation in case of ambiguity for datasets where ν_{\max} can be determined. This has already been implemented for automated analysis of *Kepler* data (Hekker et al. 2009a; Huber et al. 2009; Mathur et al. 2009). In addition, we showed that the well-used scaling relations for $\Delta\nu$ and ν_{\max} agree within a few per cent with stellar model calculations for cool models ($T_{\text{eff}} \lesssim 6400$ K), from the main sequence to the red giant branch, with a slightly increasing deviation for hotter models.

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